

## The influence of disorder on the critical sound attenuation in solids

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2005 J. Phys.: Condens. Matter 17 L485

(<http://iopscience.iop.org/0953-8984/17/46/L01>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

### Download details:

IP Address: 129.252.86.83

The article was downloaded on 28/05/2010 at 06:45

Please note that [terms and conditions apply](#).

## LETTER TO THE EDITOR

## The influence of disorder on the critical sound attenuation in solids

**P V Prudnikov and V V Prudnikov**

Department of Theoretical Physics, Omsk State University 55a, Prospekt Mira, 644077, Omsk, Russia

E-mail: [prudnikp@univer.omsk.su](mailto:prudnikp@univer.omsk.su) (P V Prudnikov)

Received 19 August 2005, in final form 7 October 2005

Published 1 November 2005

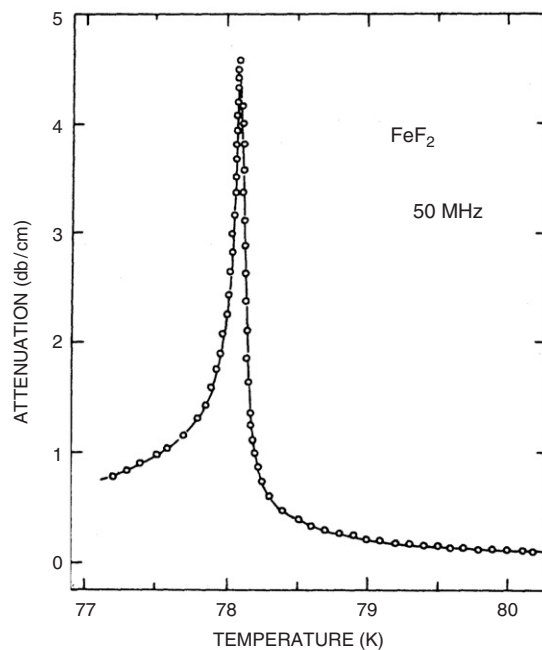
Online at [stacks.iop.org/JPhysCM/17/L485](http://stacks.iop.org/JPhysCM/17/L485)

### Abstract

The field-theoretic description of dynamical critical effects of the influence of disorder on acoustic anomalies near the temperature of the second-order phase transition is considered for three-dimensional Ising-like systems. Calculations of the sound attenuation in pure and dilute Ising-like systems near the critical point are presented. The dynamical scaling function for the critical attenuation coefficient is calculated. The influence of quenched disorder on the asymptotic behaviour of the critical ultrasonic anomalies is discussed.

The progress achieved in the understanding of critical phenomena has largely been due to theoretical and experimental works devoted to studying the critical dynamics of condensed media. We have seen from experiments (figure 1) [1] that for a solid an anomalous peak of ultrasonic attenuation is observable in the vicinity of the critical point. The critical anomalies exhibited by sound attenuation have long been recognized as important in the study of dynamical critical phenomena. Ultrasonic methods permit simultaneous measurements of both static and dynamic properties. Measurements of sound velocities give information on the equilibrium properties, while measurements of the sound attenuation yield information on the dynamic properties of materials. The main difficulty in the theoretical discussion of the critical propagation of sound waves consists in the estimation of the four-spin correlation function. The method which is based on the representation of the four-spin correlation function through two-spin correlation functions by means of decoupling leads to overestimated values of the critical fluctuations.

There are a lot of theories and phenomenological descriptions [2–9] of ultrasonic anomalies in solids with good agreement with experiments [10–12], but real materials and crystals have many structural defects and it is worth considering the influence of such defects or disorder on the dynamical process of sound propagation in solid media. Structural disorder and the presence of impurities or other defects play an important role in real materials and physical



**Figure 1.** Ultrasonic anomaly for experimental attenuation in  $\text{FeF}_2$  [1].

systems. They may induce new phase transition types and universality classes and modify the dynamic transport properties.

According to the Harris criterion [13], the critical behaviour of Ising systems is changed by the presence of a weak quenched disorder. The problem of the influence of disorder on the critical sound propagation in Ising-like systems has been discussed in [14] with use of a  $\varepsilon$ -expansion in the lowest order of approximation. However, our pilot analysis of this phenomenon showed that in [14] some diagrams which are needed for a correct description of the influence of the disorder were not considered. Furthermore, our numerous investigations of pure and disordered systems performed in the two-loop and higher orders of the approximation for the three-dimensional system directly, together with the use of methods of series summation, show that the predictions made in the lowest order of the approximation, especially on the basis of the  $\varepsilon$ -expansion, can differ strongly from the real critical behaviour [15, 16]. Therefore, the results from [14] must be reconsidered with the use of a more accurate field-theoretic approach in the higher orders of the approximation.

In this letter we have realized a correct field-theoretic description of dynamical effects of the influence of disorder on acoustic anomalies near the temperature of the second-order phase transition for three-dimensional Ising-like systems in the two-loop approximation.

In our description, we extended the model of the phase transition in a disordered system with a coupling between nonfluctuating variables [17, 18] to the case (physically important for structural phase transitions) of a compressible three-dimensional Ising model with frozen-in lattice defects that is considered in using the renormalization-group method in the two-loop approximation.

The interaction of the order parameter with elastic deformations plays a significant role in the critical behaviour of the compressible system. It was shown for the first time in [19] that the critical behaviour of a system with elastic degrees of freedom is unstable with respect to

the connection of the order parameter with acoustic modes and a first-order phase transition is realized. However, the conclusions of [19] are only valid at low pressures. It was shown in [20] that in the range of high pressures, beginning from a threshold value of pressure, the deformational effects induced by the external pressure lead to a change in type of the phase transition.

The Hamiltonian of a disordered Ising model with allowance for elastic degrees of freedom may be specified as

$$H = H_{\text{el}} + H_{\text{op}} + H_{\text{int}} + H_{\text{imp}}, \quad (1)$$

consisting of four contributions.

The elastic part is determined from

$$H_{\text{el}} = \frac{1}{2} \int d^d x \left[ C_{11}^0 \sum_{\alpha} u_{\alpha\alpha}^2 + 2C_{12}^0 \sum_{\alpha\beta} u_{\alpha\alpha} u_{\beta\beta} + 4C_{44}^0 \sum_{\alpha<\beta} u_{\alpha\beta}^2 \right], \quad (2)$$

where  $u_{\alpha\beta}(x)$  are components of the strain tensor and  $C_{ij}^k$  are the elastic moduli.

$H_{\text{op}}$  is a magnetic part in the appropriate Ginzburg–Landau form:

$$H_{\text{op}} = \int d^d x \left[ \frac{1}{2} \tau_0 S^2 + \frac{1}{2} (\nabla S)^2 + \frac{1}{4} u_0 S^4 \right], \quad (3)$$

where  $S(x)$  is the Ising field variable which is associated with the spin order parameter,  $u_0$  is a positive constant and  $\tau_0 \sim (T - T_{0c})/T_{0c}$  with the mean-field phase transition temperature  $T_{0c}$ .

The term  $H_{\text{int}}$  describes the spin–elastic interaction

$$H_{\text{int}} = \int d^d x \left[ g_0 \sum_{\alpha} u_{\alpha\alpha} S^2 \right], \quad (4)$$

which is bilinear in the spin order parameter and linear in deformations.  $g_0$  is the bare coupling constant.

The term  $H_{\text{imp}}$  of the Hamiltonian determines the influence of disorder and it is considered in the following form:

$$H_{\text{imp}} = \int d^d x \left[ \Delta\tau(x) S^2 \right] + \int d^d x \left[ h(x) \sum_{\alpha} u_{\alpha\alpha} \right], \quad (5)$$

where the random Gaussian variables  $\Delta\tau(x)$  and  $h(x)$  are the local transition temperature fluctuations and induced random stress, respectively. Taking the  $\Delta\tau(x)$  fluctuations into account causes additional interaction of order parameter fluctuations over defects and renormalization of the phase transition temperature for disordered systems. Taking into account the  $h(x)$  fluctuations leads to renormalization of the elastic moduli and the coupling constant in the spin–elastic interaction.

The Fourier transformed variables become

$$u_{\alpha\beta} = u_{\alpha\beta}^{(0)} + V^{-1/2} \sum_{q \neq 0} u_{\alpha\beta}(q) \exp(iqx), \quad (6)$$

with  $u_{\alpha\beta}(q) = i/2[q_{\alpha} u_{\beta} + q_{\beta} u_{\alpha}]$ . Then the normal-mode expansion is introduced as  $\vec{u}(q) = \sum_{\lambda} \vec{e}_{\lambda}(q) Q_{q,\lambda}$  with the normal coordinate  $Q_{q,\lambda}$  and polarization vector  $\vec{e}_{\lambda}(q)$ . We carry out the integration in the partition function with respect to the nondiagonal components of the uniform part of the deformation tensor  $u_{\alpha\beta}^{(0)}$ , which are insignificant for the critical behaviour of the system in an elastically isotropic medium.

After all of the transformations [21], the effective Hamiltonian of the system has become

$$\begin{aligned} \tilde{H} = & \frac{1}{2} \int d^d q (\tau_0 + q^2) S_q S_{-q} + \frac{1}{2} \int d^d q \Delta \tau_{-q} S_{q_1} S_{q-q_1} \\ & + \frac{1}{4} u_0 \int d^d q S_{q_1} S_{q_2} S_{q_3} S_{-q_1-q_2-q_3} + \int d^d q q h_{-q} Q_q \\ & - \frac{1}{2} w_0 \int d^d q (S_q S_{-q})(S_q S_{-q}) - g_0 \int d^d q q Q_{-q} S_{q_1} S_{q-q_1} \\ & + a \int d^d q q^2 Q_q Q_{-q}. \end{aligned} \quad (7)$$

The renormalization-group analysis of the critical behaviour of the disordered compressed Ising model with the Hamiltonian (7) was carried out in our paper [21], and different fixed points for the Hamiltonian (7) and conditions of their stability were determined in the two-loop order of the approximation with the use of the Padé–Borel summation technique. It was shown that the  $\Delta \tau_q$  fluctuations are relevant for the critical behaviour of the Ising model and the replica averaging procedure over  $\Delta \tau_q$  leads to a new vertex of interaction for the order parameter fluctuations. It was shown in [21] that random stresses connected with  $h_q$  in (7) can lead to multicritical behaviour of a system if certain conditions are fulfilled.

The critical dynamics of the system in the relaxational regime can be described by the Langevin equations [22] for the spin order parameter  $S(q)$  and phonon normal coordinates  $Q(q)$ :

$$\begin{aligned} \dot{S}_q &= -\Gamma_0 \frac{\partial \tilde{H}}{\partial S_{-q}} + \xi_q + \Gamma_0 h_S, \\ \dot{Q}_q &= -\frac{\partial \tilde{H}}{\partial Q_{-q}} - q^2 D_0 \dot{Q}_q + \eta_q + h_Q, \end{aligned} \quad (8)$$

where  $\Gamma_0$  and  $D_0$  are the initial kinetic coefficients,  $\xi_q(x, t)$  and  $\eta_q(x, t)$  are Gaussian white noises.

The quantities of interest are the response functions  $G(q, \omega)$  and  $D(q, \omega)$  of the spin and deformation variables, respectively. It can be obtained by linearization in corresponding fields that

$$D(q, \omega) = \delta [\langle Q_{q, \omega} \rangle] / \delta h_Q = [\langle Q_{q, \omega} Q_{-q, -\omega} \rangle], \quad (9)$$

$$G(q, \omega) = \delta [\langle S_{q, \omega} \rangle] / \delta h_S = [\langle S_{q, \omega} S_{-q, -\omega} \rangle], \quad (10)$$

where  $\langle \dots \rangle$  denotes averaging over Gaussian white noises,  $[\dots]$  denotes averaging over random fields  $\Delta r_q$  and  $h_{-q}$ .

The response functions may be expressed in terms of self-energy parts:

$$\begin{aligned} G^{-1}(q, \omega) &= G_0^{-1}(q, \omega) + \Pi(q, \omega), \\ D^{-1}(q, \omega) &= D_0^{-1}(q, \omega) + \Sigma(q, \omega), \end{aligned} \quad (11)$$

where the free response functions  $G_0(q, \omega)$  and  $D_0(q, \omega)$  have the forms

$$\begin{aligned} D_0(q, \omega, \lambda) &= 1 / (\omega^2 - a q^2 - i \omega D_0 q^2), \\ G_0(q, \omega) &= 1 / (i \omega / \Gamma_0 + (\tau_0 + q^2)). \end{aligned}$$

The characteristics of the critical sound propagation are defined by means of the response function  $D(q, \omega)$ . Thus, the coefficient of ultrasonic attenuation is determined through the imaginary part of  $\Sigma(q, \omega)$ :

$$\alpha(\omega, \tau) \sim \omega \operatorname{Im} \Sigma(\omega). \quad (12)$$

$$\begin{aligned}
 \Sigma(q, \omega) = & 4g^2 \text{ (loop with cross)} - 96g^2u \text{ (two loops with cross)} + \\
 & + 16g^2v \text{ (loop with cross and dashed)} + 16g^2v \text{ (loop with cross and dashed)} + 16g^2v \text{ (loop with cross and dashed)} + \\
 & + 16g^2v \text{ (loop with cross and dashed)} + 16g^2v \text{ (loop with cross and dashed)} + 16g^2v \text{ (loop with cross and dashed)}
 \end{aligned}$$

(a) 
(b) 
(c)

**Figure 2.** The diagrammatic representation of the  $\Sigma(q, \omega)$  in the two-loop approximation. Solid line (a) corresponds to  $G_0(q, \omega)$ , solid line with a cross (b) is  $2G_0(q, \omega)G_0(-q, -\omega)/\Gamma_0$  and vertex with a dashed line (c) corresponds to  $v = [(\Delta\tau)^2]$ .

The self-energy part  $\Sigma(q, \omega)$  was obtained by iterative solution [23] of the dynamic equation (8) with the effective Hamiltonian (7). The diagrammatic representation of  $\Sigma(q, \omega)$  in the two-loop approximation is presented in figure 2.

The Feynman diagrams involve momentum integrations in dimension  $d$  (in our case  $d = 3$ ). Near the critical point the correlation length  $\xi$  increases infinitely. When  $\xi^{-1} \ll \Lambda$ , where  $\Lambda$  is a cut-off in the momentum-space integrals (the cut-off  $\Lambda$  serves to specify the basic length scale), the vertex functions are expected to display an asymptotic scaling behaviour for wavenumbers  $q \ll \Lambda$ . Therefore, one is led to consider the vertex functions in the limit  $\Lambda \rightarrow \infty$ . The use of the renormalization-group scheme removes all divergences which arise in thermodynamic variables and kinetic coefficients in this limit.

We have applied the matching method which was introduced for statics in [24] and generalized for critical dynamics in [4]. First, we use the dynamical scaling property of the response function

$$D(q, \omega, \tau) = e^{2l} D(qe^l, (\omega/\Gamma_0)e^{zl}, \tau e^{l/\nu}), \tag{13}$$

and then we calculate the right-hand side of this equation for some value  $l^* = l$ , where the arguments do not all vanish simultaneously [24]. The choice of  $l^*$  is determined by

$$(\omega_l/2\Gamma_0)^{4/z} + \chi_l^{-2} = 1. \tag{14}$$

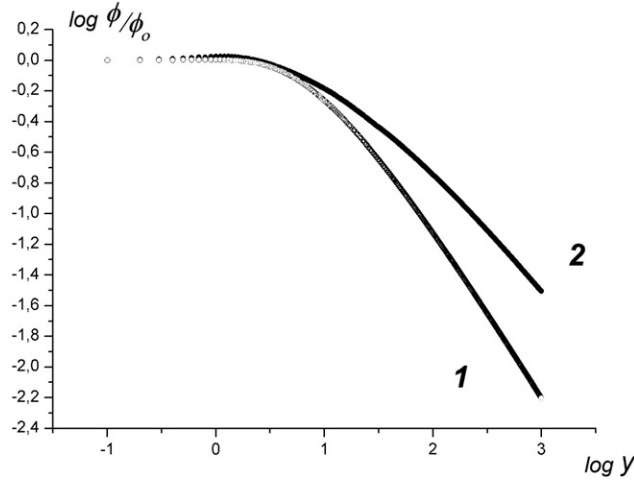
Equation (13) guarantees that at least one of the arguments of the right-hand side of (14) is finite. The particular form of the matching condition (14) containing the exponents  $z$  and  $\nu$  permits an explicit solution for  $l^*$ :

$$e^l = \tau^{-\nu} [1 + (y/2)^{4/z}]^{-1/4} \equiv \tau^{-\nu} F(y, \tau). \tag{15}$$

In (15) the abbreviation  $y = \omega\tau^{-z\nu}/\Gamma_0$  is introduced and  $F(y, \tau)$  is defined. The value of the exponent  $\nu$  was calculated in [21] for the corresponding fixed point. The exponent  $z$  was taken from [25] for the 3D disordered Ising model considered for purely relaxational model A. The coupling of the order parameter with elastic deformations is irrelevant for the dynamics of a disordered system with a negative specific heat exponent [26].

The response function  $D(qe^l, (\omega/\Gamma_0)e^{zl}, \tau e^{l/\nu})$  on the right-hand side of (13) is represented by the Dyson equation (11) and for the self-energy part we obtain

$$\frac{\text{Im } \Sigma(\omega)}{\omega} = \exp\left(l \frac{(\alpha + z\nu)}{\nu}\right) \frac{\text{Im } \Sigma(\omega e^{zl})}{\omega e^{zl}}. \tag{16}$$



**Figure 3.** Scaling functions for the critical sound attenuation  $\phi(y)$  in a double-logarithmic plot for pure (1) and disordered (2) Ising systems.

It may be argued [4] that condition (14) together with the well-known expression for the susceptibility [14] provides an infrared cut-off for all divergent values.

It was shown in later theoretical works [3, 12] that in asymptotic regions the coefficient of attenuation can be described using a simple scaling function of the variable  $y$  only. The experimental investigations performed on the three-dimensional crystals Gd [11] and MnP [12] confirmed the validity of the concepts of dynamical scaling.

Thus after renormalization procedure (16) we can define a scaling relation in the form

$$\text{Im } \Sigma(\omega)/\omega = \tau^{-\alpha-z\nu} \phi(y), \quad (17)$$

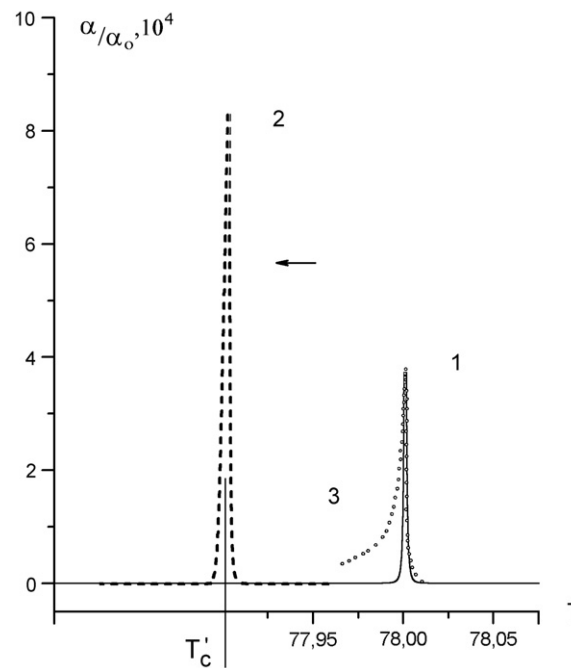
where  $\phi(y)$  is a dynamical scaling function:

$$\begin{aligned} \phi(y) = & \frac{g^{*2}\Gamma_0}{\pi} \frac{F^{\alpha/\nu+1/2\nu-z}}{y^2} \left\{ 1 - \left[ \frac{1}{2} \left( 1 + \frac{y^2 F^{2z-2/\nu}}{4} \right)^{1/2} + \frac{1}{2} \right]^{1/2} \right\} \\ & - \frac{12g^{*2}u^*\Gamma_0^2}{\pi^2} \frac{F^{\alpha/\nu+1/\nu-2z}}{y^3} \left\{ \left[ \frac{1}{2} \left( 1 + \frac{y^2 F^{2z-2/\nu}}{4} \right)^{1/2} - \frac{1}{2} \right]^{1/2} - \frac{y F^{z-1/\nu}}{4} \right\} \\ & + \frac{8g^{*2}v^*}{(4\pi)^3} \frac{F^{\alpha/\nu-z}}{y^2} \phi_{\text{imp}}(y), \end{aligned} \quad (18)$$

where  $g^*$ ,  $u^*$  and  $v^*$  are values of vertices in the fixed point of renormalization group transformations [21],  $\phi_{\text{imp}}(y)$  is the numerically calculated contribution of diagrams in  $\Sigma(\omega)$  (figure 2) characterizing the influence of disorder.

The dynamical scaling function  $\phi(y)$  is plotted against  $y$  on a double-logarithmic scale for pure and disordered systems in figure 3. We have thus seen that the presence of disorder is irrelevant for the scaling behaviour in the hydrodynamic region with  $y \ll 1$  ( $T \gg T_c$ ), but it has a drastic effect in the critical region with  $y \gg 1$  ( $T \rightarrow T_c$ ).

The anomalous temperature dependence of the calculated attenuation coefficient for pure and disordered systems is shown in figure 4. Comparison of curves (1) and (2) in figure 4 clearly reveals the strong influence of disorder on the temperature dependence of the attenuation coefficient in the vicinity of the critical point. We hope that these theoretical results will create



**Figure 4.** Thermal dependences of the ultrasonic attenuation at the critical point for pure (1) and disordered (2) systems in comparison with experimental results (3) for FeF<sub>2</sub> [1].

**Table 1.** Asymptotic behaviour of the attenuation coefficient in the hydrodynamic ( $T \gg T_c$ ) and critical ( $T \rightarrow T_c$ ) regions.

System	Region	$\alpha(\omega, \tau)$
Pure	$T \gg T_c$	$\omega^2 \tau^{-1.38}$
	$T \rightarrow T_c$	$\omega^{0.914}$
Disordered	$T \gg T_c$	$\omega^{2.02} \tau^{-1.47}$
	$T \rightarrow T_c$	$\omega^{1.23} \tau^{-0.26}$
Pure [5]	$T \gg T_c$	$\omega^{2.03} \tau^{-1.33}$
	$T \rightarrow T_c$	$\omega^{0.913}$

a demand for ultrasonic experimental investigations of dilute Ising-like systems, for example samples of Fe<sub>x</sub>Zn<sub>1-x</sub>F<sub>2</sub>. In producing the figure we used the fact that the presence of disorder causes a reduction of the phase transition temperature  $T_c$  in relation to that of the pure system. A model representation of the calculated attenuation coefficient for the pure system is shown in figure 4 in comparison with experimental data (3) for FeF<sub>2</sub> [1]. Adjustment of the experimental data permitted us to determine the value of the theoretical parameter  $\Gamma_0$  and then calculate the attenuation coefficient for the disorder model. We should note that the observable differences from the experimental results below  $T_c$  are explained by the contribution of the order parameter relaxation effects to the attenuation, which always occur below  $T_c$ . The relaxation effects are not considered in this letter, but the contribution of fluctuations to the attenuation coefficient is relevant over the whole critical range.

From (12) we find that the sound attenuation coefficient obeys the asymptotic scaling relation

$$\alpha(\omega, \tau) \sim \omega^2 \tau^{-\alpha-\nu\tau} \phi(y). \quad (19)$$



On the basis of the dependences (18) obtained, we calculated the exponents of the asymptotic behaviour of the attenuation coefficient in the hydrodynamic and critical regions (table 1). We see from the table that in the critical region the anomalies of the attenuation coefficient must be observed in both pure and disordered systems. But for disordered systems the anomalies of the temperature and frequency dependences of the attenuation coefficient must be stronger than those for pure systems. These conclusions are also shown in figure 4.

We hope that theoretical estimates of the attenuation coefficient carried out in this paper create a great demand for detection of the anomalies which are found to be induced by quenched disorder in ultrasonic experimental investigations of critical dynamics.

This work was supported by the Russian Foundation for Basic Research through Grants (Nos 04-02-17524, 04-02-39000 and 05-02-16188) and the Ministry of Education and Science of Russia through Grant No UR.01.01.230.

## References

- [1] Ikushima A and Feigelson R 1971 *J. Phys. Chem. Solids* **32** 417
- [2] Landau L D and Khalatnikov I M 1965 *Collected Papers of Landau* (New York: Pergamon)
- [3] Kawasaki K 1976 *Phase Transitions and Critical Phenomena* vol 5a (London: Academic) p 165  
Kawasaki K 1977 *Proc. Conf. on Internal Friction and Ultrasonic Attenuation in Solids* (Tokyo: Tokyo University Press) p 29
- [4] Folk R, Iro H and Schwabl F 1977 *Z. Phys. B* **27** 169  
Dengler R, Iro H and Schwabl F 1985 *Phys. Lett. A* **111** 121
- [5] Schwabl F and Iro H 1981 *Ferroelectrics* **35** 215  
Iro H and Schwabl F 1983 *Solid State Commun.* **46** 205  
Dengler R and Schwabl F 1987 *Z. Phys. B* **69** 327
- [6] Pawlak A 1989 *J. Phys.: Condens. Matter* **1** 7989  
Pawlak A 1991 *Phys. Rev. B* **44** 5296
- [7] Schorgg A M and Schwabl F 1993 *Phys. Rev. B* **49** 11682
- [8] Kamilov I K and Aliev H K 1998 *Phys. Usp.* **41** 865
- [9] Aliev H K *et al* 1989 *Sov. Phys.—JETP* **95** 1896
- [10] Bhattacharjee J K 1982 *Phys. Rev. B* **25** 3404  
Ferrel R A, Mirhashem B and Bhattacharjee J K 1987 *Phys. Rev. B* **35** 4662
- [11] Lüthi B, Moran T J and Pollina R G 1970 *J. Phys. Chem. Solids* **31** 1741  
Moran T J and Lüthi B 1971 *Phys. Rev. B* **4** 122
- [12] Suzuki M and Komatsubara T 1982 *J. Phys. C: Solid State Phys.* **15** 4559
- [13] Harris A B 1974 *J. Phys. C: Solid State Phys.* **7** 1671
- [14] Pawlak A and Fechner B 1989 *Phys. Rev. B* **40** 9324
- [15] Prudnikov V V, Prudnikov P V and Fedorenko A A 2000 *Phys. Rev. B* **62** 8777  
Prudnikov V V, Prudnikov P V and Fedorenko A A 2001 *Phys. Rev. B* **63** 184201
- [16] Pelissetto A and Vicari E 2002 *Phys. Rep.* **368** 549  
Folk R, Holovatch Yu and Yavors'kii T 2003 *Phys. Usp.* **46** 169
- [17] Laptev V M and Skryabin Yu N 1979 *Phys. Status Solidi b* **91** K143
- [18] Skryabin Yu N and Shchanov A V *Phys. Lett. A* 234147
- [19] Larkin A I and Pikin S A 1969 *Sov. Phys.—JETP* **29** 891
- [20] Imry Y 1974 *Phys. Rev. Lett.* **33** 1304
- [21] Prudnikov V V and Belim V S 2001 *Phys. Solid State* **43** 1353
- [22] Meissner G 1980 *Ferroelectrics* **24** 27  
Schwabl F and Iro H 1981 *Ferroelectrics* **35** 27
- [23] Ma Sh 1976 *Modern Theory of Critical Phenomena* (London: Benjamin)
- [24] Nelson D R 1976 *Phys. Rev. B* **14** 1123
- [25] Prudnikov V V *et al* 1998 *JETP* **87** 527
- [26] Hohenberg P C and Halperin B I 1977 *Rev. Mod. Phys.* **49** 435